Diagrammatic activity and communicating about it in individual learning support: Patterns and dealing with errors

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Diagrammatic activity and communicating about it in different representational systems is essential for mathematical discourse. Therefore, it is also of great importance for individual mathematical support. So far, little is known about how teachers conduct such discourses with children. By analyzing the diagrammatic activities and the communication about them with an analyzing method developed for this purpose, patterns of diagrammatic activity and communicating about it can be elaborated. The example of an individual learning support between a preservice teacher and a second grade student who work in the representational systems of natural numbers and the field of twenty is presented. It shows that these patterns can be supportive in dealing with errors.

Keywords: Individual learning support, diagrammatic activity, mathematical discourses, arithmetics.

Individual learning support as increasing participation in mathematical discourse

When learning mathematics, not all students achieve the teaching goals through joint teaching in the class. These students need individual learning support. Individual learning support can be understood as "support of individual […] learners by a person with more expertise in building knowledge or acquiring competences to solve a task or a problem with a view to the learners' future independent mastering of analogous tasks and problems" (Krammer, 2009, p. 89; translated by the authors). This understanding of the term is closely related to Vygosky's concept of the zone of proximal development (Vygotsky, 1978). In this and in concepts based on it (e.g. Rogoff, 1990), the importance of the social and thus of interaction for cognitive development is emphasized. According to Sfard (2001), learning mathematics can thus be understood as a cultural practice. Thus, learners do not encounter mathematics per se, but a cultural practice that is understood as mathematical by members of that culture (see also Tiedemann, 2012). In this sense, learning mathematics means being able to participate increasingly confidently in discourse that is considered mathematical by experts (Sfard, 2001). Interaction with experts is essential for this. In such an interactionist view of learning (Krummheuer, 1992), mathematical support can be understood "as a specific process of interaction […] that is successively established [by adult and child] through a mutually interrelated interpretation and action" (Tiedemann, 2012, p. 49f.; translated by the authors). According to Dörfler (2006), "progressive participation in the social practice of diagrammatic activities" can be seen as typical of mathematical discourse and mathematical cultural practice (p. 213; translated by the authors). Thus, individual learning support in mathematics means that learners gain expertise in diagrammatic activities and can thus participate in mathematical discourse. Tiedemann (2012) was able to identify general patterns of interaction between parents and children in early mathematical education. However, there is a lack of research on what such interaction patterns and routines look like in individual learning support, especially with regard to diagrammatic activities.

Diagrammatic activity and communicating about it

Diagrams, understood in the sense of the American philosopher Charles Sanders Peirce (1839-1914), are seen in this paper as objects of mathematical activity. In this sense, diagrams are signs with a relational character that belong to a representational system (Dörfler, 2006). The representational system determines how diagrams are created and how they are operated or experimented with. For students, each representational system is its own learning content, as is each direction in moving between different representational systems. In what follows, activities with diagrams that are specified by a representational system are referred to as diagrammatic activities (Wille, 2020). Gestures can also be part of a diagram as quasi-materialized inscriptions (Huth, 2020) and likewise be part of diagrammatic activities. It should be noted that a diagram can be interpreted as such if a corresponding representational system is known (Wille, 2020). That is, no diagram is in itself a diagram. It requires interpretation as one. Activities with diagrams such as experimenting or observing can help clarify, structure, and coordinate thought processes (Hoffmann, 2007). In addition, diagrammatic activities can give rise to mathematical concepts and understanding (Dörfler, 2006). In this way, diagrams themselves become the subject of reasoning processes. Another essential part of mathematical activity is communication about it (Dörfler, 2006). Communication about diagrams and diagrammatic activity involves both linguistic and gestural utterances. Communication about diagrammatic activity enables the use of denotations for diagrams that belong to different representational systems, and it also enables interpretations of diagrammatic thinking (Wille, 2020).

For learning processes, it is crucial that students do not just do something, but that their attention is drawn to the crucial aspects through communication (Gaidoschik, 2016). It is often reported that teachers have few professional discourses with the learners and do not pay much attention to the learning processes of the students in the dialogues, but rather often dominate the classroom discussion (Krammer, 2009; Begehr, 2006). Consequently, it is necessary to already qualify preservice teachers to conduct such professional discourses with learners. However, there is a lack of research on how diagrammatic activities and communication about them intertwine in individual learning support between a preservice teacher and a student.

Research interest

The research focuses on situations of individual learning support between a preservice teacher and a student in grade 1 or 2. It addresses the question of how diagrammatic activity and the communication about it intertwine in individual learning support. Since support is most evident when learners have particular difficulties or make an error, special attention is paid to how diagrammatic activity and the communication about it intertwine in such situations.

Setting

At the St. Gallen University of Teacher Education, preservice teachers provide one-on-one support to first and second grade children as part of an elective course. In the spring semester, one pre-service teacher supports one child per week for about 30 minutes. A seminar at the university accompanies the support sessions, which in particular are recorded on video. In the individual learning support, the preservice teachers make use of support activities that were developed in the MALKA project (Wehren-Müller et al., 2018). The goal of individual learning support is to enable students to solve arithmetic tasks without counting strategies, because a solidification of initial counting strategies can lead to difficulties in learning mathematics (Scherer & Moser Opitz, 2010). Due to this, it is necessary for students to develop sustainable ideas about numbers and operations (Häsel-Weide, 2016). To achieve this, learners should be supported in perceiving and determining the cardinality of a quantity through structural subitizing (Schöner & Benz, 2018). This can be practiced especially on structured materials, such as the field of twenty (Häsel-Weide, 2016; Scherer & Moser Opitz, 2010). The representational systems that are essential for structural subitizing in the individual learning support analyzed here are the field of twenty and the natural numbers.

Method

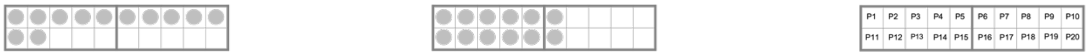
The analysis takes place in several steps. In a first step, an interaction analysis is carried out to reconstruct the interaction processes in detail (Krummheuer & Naujok, 1999). In a second step, the diagrammatic activity and the communication about it is analyzed. For this purpose, an analysis method developed by Wille (2020) for imagined dialogues was adapted for interactions in the two representation systems, the field of twenty and the natural numbers (Ott & Wille, 2021). An analysis sheet for the whole episode is filled in according to the following rules:

- If a *diagram* is used in a turn, a *filled circle* is set in the column of the corresponding representational system. If *communication about* diagrams is used, a *dashed circle line* is set. If both take place, both are noted together. A star is used if an activity cannot be interpreted as a diagram or communicating about it.
- The filled circles or dashed circle lines are connected to each other by *solid lines* if a connection is made by *diagrammatic activities*. The line is *dashed* when the connection is made by *communicating about diagrams*. If both occur, both are noted together.
- If, in a turn, *diagrams of different representational systems correspond* with each other, they are connected by an *arrow*. The direction of the arrow indicates which representational system is used as the starting point.
- If *diagrams that have already occurred once occur again in exactly the same way*, they are connected by *two narrow lines*.
- Communication that cannot be assigned to either one or the other representation system is noted as “others”.

In the following, the results of the analysis of diagrammatic activity and the communication about it of one preservice teacher and one child will be presented. Activities of the preservice teacher are noted in red, activities of the child in blue.

Results

In the following, we will first show some patterns that become visible in the analysis sheet. Afterwards, the handling of errors will be described. The scenes shown in the following are taken from two support episodes between the preservice teacher Mr. Wehrle, who is in the fifth semester of six semesters, and Samira, who is a student at the beginning of the second grade. Mr. Wehrle and Samira work on a structural subitizing task, in which the number of chips in different arrangements on the field of twenty are to be determined (Scherer & Moser Opitz, 2010) (see Figure 1a and 1b). They work with chips and with strips of five on the field of twenty. In the transcript, the field of twenty is numbered as shown in Figure 1c. The transcripts were originally in German and were more detailed.



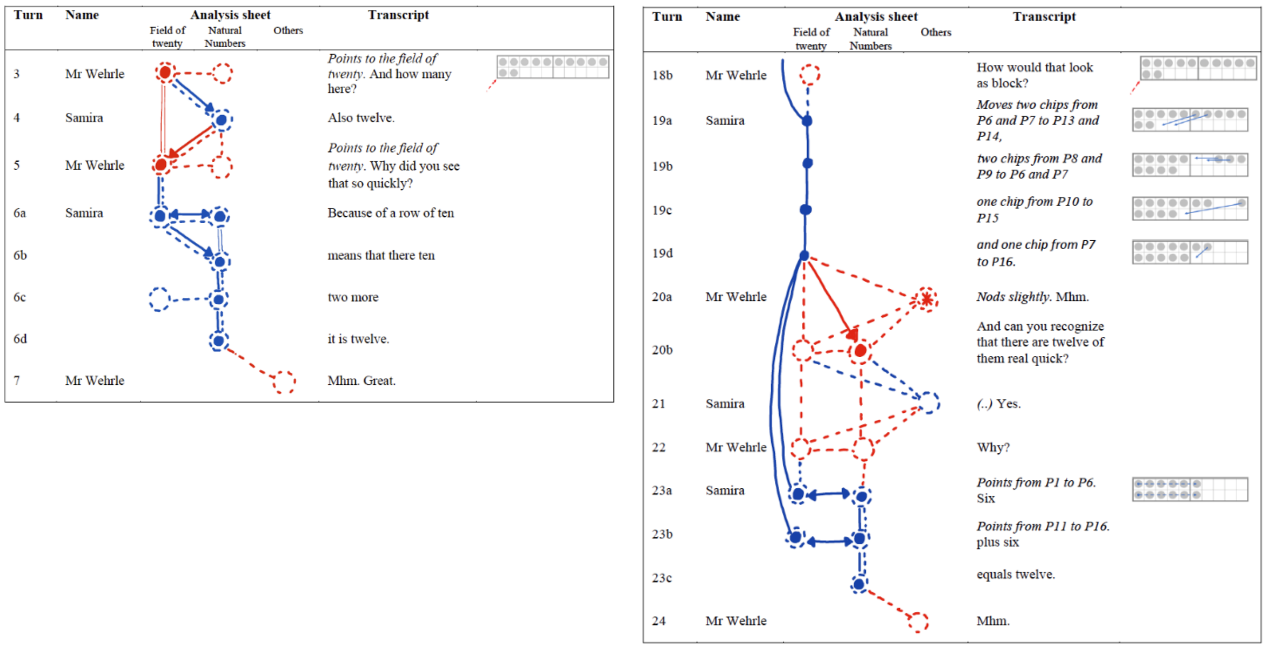
a b c

Figure 1: Field of twenty, (a) row arrangement (b) block arrangement (c) numbering in the transcript

Samira initially has difficulty determining numbers by structural subitizing when they are arranged in a block (see Figure 1b), especially with odd numbers. She becomes increasingly confident in this, but she repeatedly makes errors in determining odd numbers.

Patterns

The analysis sheets show some patterns in the interaction regarding diagrammatic activities and communicating about them. The two selected scenes from episodes 1 and 2 (see Figure 2 and 3) exemplify these patterns.



a b

Figure 2: Analysis sheets and transcripts, (a) episode 1, scene 1 (b) episode 2, scene 3

- Mr. Wehrle sets the tasks for Samira often in such a way that both representational systems are linked at least by communicating about a diagram (see Turn 3 and 5 in Figure 2a; Turn 20b in Figure 2b). Samira then also combines both representational systems in her answers (see Turn 6a in Figure 2a; Turn 23a, b in Figure 2b) by using diagrams in both representational systems.
- Mr. Wehrle promts Samira repeatedly to switch back and forth between the representational systems. To this end, he himself switches between the representational systems. He adapts this to the respective situation. If Samira's answer is only in the natural numbers, he switches to the field of twenty (see Turn 4–5 in Figure 2a). If her answer is only in the field of twenty,

he switches to the natural numbers (see Turn 19d–20b in Figur 2b). Therefore, he uses corresponding diagrams to the ones Samira used in the other representational system.

- Diagrams are mainly used by Samira. She also mainly carries out the diagrammatic activities. Mr. Wehrle encourages these according to the principle of minimal help (Aebli, 1998). The impulse to communicate about diagrams usually comes from Mr. Wehrle. He almost always communicates via the diagrams he uses.

Dealing with errors

If a content-related error has occurred during the task processing, a correction is needed. Samira repeatedly makes errors in determining odd numbers that are arranged in a block. In the analysis sheets, errors are indicated with a flash. Figure 3 shows three analysis sheets for error situations.

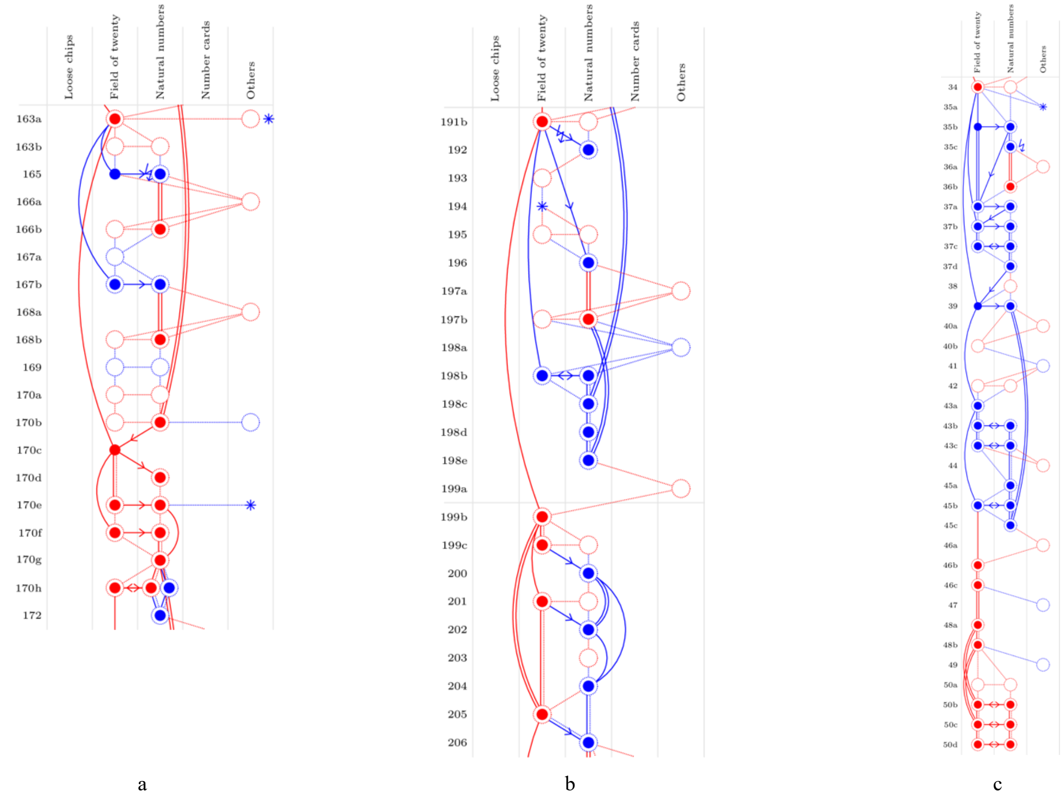
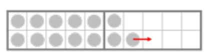
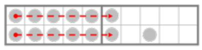
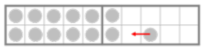


Figure 3: Analysis sheets of error situations, (a) episode 1, scene 13 (b) episode 1, scene 15 (c) episode 2, scene 5

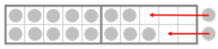
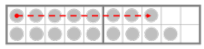
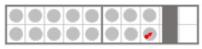
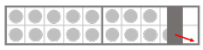
In episode 1, scene 13 (Figure 3a), Samira determines an incorrect number and corrects herself when Mr. Wehrle asks where she sees this number (T. 164–168a). He then asks her, how she got the correct number and Samira reports that she counted (T. 168–170a). Now Mr. Wehrle becomes diagrammatically active himself to correct the error by referring to a known procedure and carrying it out himself. In doing so, he always connects the two sign systems with each other and communicates via each diagram. He thus demonstrates to Samira the possibility of non-counting number determination by way of example. Samira participates by agreeing and taking over the diagrammatic activity at the end:

170b Mr. Wehrle: You had a great trick at twelve earlier.
170c *Pushes the chip from P17 to P18.*



170d At twelve you said
170e here are six. *Points from P1 to P6.* < 
170f And here are six. *Points from P11 to P16.* <<
170g Equals twelve.
170h *Pushes the chip back from P18 to P17.* 
And then <<< only plus one.
171a Samira: < Yes.
171b << *Nods.*
171c <<< One.
172 Samira: Equals thirteen.

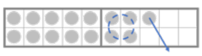
In episode 1, scene 15 (Figure 3b), Samira determines an incorrect number, too. With Mr. Wehrle's advice to look carefully and count the top row, she is able to give the correct number (T. 191–196). When asked how she figured it out, she refers to the previous number and states that there is now one more chip (T. 197b–198e). She is mainly diagrammatically active in the natural numbers. Mr. Wehrle then works with her to determine the number by structural subitizing, always combining both representational systems. He takes the field of twenty as the starting point and encourages Samira to switch between the sign systems. He takes over the diagrammatic activities at the field of twenty:

199b Mr. Wehrle: And have a look. *Puts two chips on P8 and P19.* 
199c *Points along the top row of chips.* If you now count this row up here. How many are there? 
200 Samira: *(4 sec.)* Eight.
201 Mr. Wehrle: *Covers the chip on P19.* How many are there then in the bottom row up to here? *Points to P18.* 
202 Samira: Eight.
203 Mr. Wehrle: Equals
204 Samira: Sixteen.
205 Mr. Wehrle: *Uncovers the chip on P19.* And this one. 
206 Samira: Seventeen.

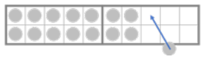
In episode 2, scene 5 (Figure 3c), again, Samira determines an incorrect number (T. 34–35). When asked why there are this number of chips, Samira now becomes diagrammatically active. She uses both representational systems flexibly, switches back and forth between them and can identify her answer as incorrect and name the correct number:

37a Samira: *Points from P1 to P8.* I counted here, that—it's eight. 
37b *Points from P11 to P17.* But there, it can't be eight.
37c *Points to P18.* Because there should be one more.
37d So, it is not sixteen.
38 Mr. Wehrle: *(..)* But?
39 Samira: *(10 sec.) Makes slight nodding movements with the head.*
Fifteen.

Mr. Wehrle asks her not to count the number. Thereupon Samira again becomes diagrammatically active. Again, she uses both representational systems flexibly, switching back and forth between them. Now, however, she is proceeding in a very orderly manner:

43a Samira: *(.)* For example. *Pushes the chip from P8 away next to the field of twenty.* 
43b Now there are *(.)* seven. *Slight circular movement via P6, P7, P8, P9.*
43c Plus seven.
44 Mr. Wehrle: Mhm.

45a Samira: Equals fourteen,
45b and one more *Pushes the chip back to P8.*
45c equals fifteen



Mr. Wehrle repeats this approach and enriches it with terms such as “doubling” (T. 46–49).

Overall, development can be seen: while Mr. Wehrle initially demonstrates a non-counting strategy, he increasingly takes a step back and challenges Samira to diagrammatic activities. Conversely, Samira is initially strongly focused on the natural numbers. Increasingly, she connects them with the field of twenty and finally becomes independently active in both representational systems in order to correct her error. While Samira initially has to be prompted to communicate about the diagrams and the diagrammatic activity, in scene 5 (episode 2) she does this on her own and uses it to think aloud.

Conclusions and limitations

By analyzing the diagrammatic activity and communicating about it in individual support episodes as a whole, patterns and developments become apparent. In the individual learning support presented here, for example, these patterns can be seen in the fact that Samira was repeatedly encouraged to switch between the two representational systems and to connect them. In this way, Samira can increasingly participate in the mathematical practice of diagrammatic activities (Dörfler, 2006). Mr. Wehrle uses the diagrams of different representional systems, mainly of the field of twenty and the natural numbers, as the subject of the reasoning and communication processes (Dörfler, 2006). In doing so, he encourages Samira to explore and interpret these diagrams. This interaction leads to Samira being able to clarify and structure and her thought processes (Hoffmann, 2007). Especially when dealing with errors, this becomes evident: Samira is finally able to engage in a mathematical discourse for error correction using diagrammatic activity and communicating about. In doing so, she applies elements of the patterns that were previously established between her and Mr. Wehrle.

This study has some limitations up to this point. Since only one situation of individual learning support was initially considered, it is yet unclear whether such a pattern also leads to a similar result in other situations and whether success might be found in other patterns of interaction. In future research, individual learning support of different preservice teachers will be compared in order to obtain insights into teacher training in this area in the long run.

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