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AN ANALYSIS OF DIAGRAMMATIC ACTIVITY AND COMMUNICATING ABOUT IT IN INDIVIDUAL LEARNING SUPPORT

Barbara Ott*, Annika M. Wille**

*St. Gallen University of Teacher Education, Switzerland

**University of Klagenfurt, Austria

Preservice teachers need to learn how to support children with learning difficulties. This requires a critical perspective on one's own teaching. In this paper a method is presented in order to analyse how preservice teachers support elementary school students who mainly use counting strategies when solving arithmetic problems. Using this analysing method, it becomes visible what mathematical sign activity takes place during the support and how it is intertwined with the communication about it. In this way patterns become visible in how a preservice teacher supports a child and how the child reacts. Finally, the resulting visualisation can serve as a basis for preservice teachers to reflect on their teaching.

INTRODUCTION

For learning and doing mathematics, activity with signs is necessary. From the beginning, students have to learn how to use different representational systems and how to combine them. Among other things, grade 1 students need to learn to solve arithmetic problems without counting strategies. The counting strategies are first elementary approaches, but a solidification of them can lead to difficulties in learning mathematics (Scherer & Moser Opitz, 2010). To counteract this, it is important, that learners develop a structure sense (Lüken, 2012). For this purpose, mathematical tools are used, such as the twenty field (see Figure 1). In this case, in addition to learn how to use the representational system of natural numbers, the children must also learn how to use the representational system twenty field and make connections between the two in both directions. To achieve this, some children need special support. In order to enable future teachers to provide such assistance, at the St. Gallen University of Teacher Education, the individual support of children to learn to solve arithmetic problems without counting strategies is already taken into account during the teacher education. The associated project *MaL1zu1 - Learning and Teaching Mathematics in One-to-One Support*, which is a subproject of the of the project *MALKA - Learning and Cooperating in Mathematics from the very Beginning* (PHSG, 2018), investigates how preservice teachers support grade 1 and 2 students in an individual setting. An analysis of the supportive interactions will provide information on how sign activity is induced in children by the preservice teachers and which interactional patterns arise with regard to the sign activity in

different representational systems. The long-term goal is to use these visualisations as a basis for reflective discussions with preservice teachers about their teaching. In this article the method of analysis is presented by means of three exemplary scenes.

THEORETICAL FRAMEWORK

Solving arithmetical problems without counting strategies

In order to enable children to solve arithmetical problems without counting strategies, they must be supported in the development of sustainable ideas about numbers and operations (Häsel-Weide, 2016). In this context, it is particularly important that the children develop the part-whole-schema (Gerster & Schultz, 2004). Therefore, the children should be supported in recognizing numbers as structured quantities in combination with decomposing, representing and describing them (Häsel-Weide, 2016, p. 32). Thus, the learners should be able to perceive and determine cardinality of a quantity by structural subitizing (Schöner & Benz, 2018). That means that the quantities are perceived in structures and that the determination of the quantity is based on known facts without using counting strategies (Schöner & Benz, 2018, p. 127). Structured materials such as the twenty field are suitable for this (Häsel-Weide, 2016; Scherer & Moser Opitz, 2010).

Diagrammatic Activity and Communicating about it

In mathematics, diagrammatic inscriptions are of particular importance. In this article, *diagrams* will be considered from the perspective of the American philosopher Charles Sanders Peirce (1839-1914) as signs with a relational character, whose perceptible basis is an inscription (Dörfler, 2008). Several characteristics qualify an inscription as a diagram (Dörfler, 2016). A main characteristic of diagrams is that they are not individual, isolated inscriptions, but belong to a representational system. Thus, there are certain means and rules for their creation, reading and use. In the following, these activities with diagrams given by a representational system are called *diagrammatic activities* (Wille, 2020). Gestures as quasi-materialized inscriptions can be part of a diagram (Huth, in press). Thus, gesturing can also be part of diagrammatic activities. However, no diagram is a diagram by itself, but can be interpreted as such, if an appropriate representational system is known (Wille, 2020). Activities like constructing, experimenting, observing, noting, and assuring with the inscriptions help to clarify, structure, and coordinate thinking processes (Hoffmann, 2007). Thus, diagrammatic inscriptions themselves become the objects of argumentation processes. Furthermore, communication about them is possible (Dörfler, 2008). *Communication about diagrams and diagrammatic activity* includes both spoken and gestural expressions. As sign activity itself, communicating about it is an inevitable part of mathematical activity. It provides the use of denotations for diagrams that belong to different representational

systems and in addition interpretations of diagrammatic reasoning (Wille, 2020). Furthermore, communicating about sign activity can lead to reflection. Reflection can be understood as a change of position (Freudenthal, 1991). This enables reinterpretations or the adoption of the perspectives of others. It can be caused, for example, by moments of irritation (Schülke, 2013) and can lead to new diagrammatic activities or a different interpretation of the diagram.

RESEARCH INTEREST

How does a preservice teacher support a child to overcome counting strategies for solving arithmetical problems and, within this support, how do diagrammatic activity and the communication about it intertwine?

SETTING

In an elective subject, preservice teachers support children of the first and second grade in learning to solve arithmetical problems without counting strategies in an individual support: One preservice teacher supports one child approximately 30 minutes per week during the spring semester. They work with support activities that were developed in the MALKA project (Wehren-Müller et al., 2018). At the beginning and at the end of the semester the competencies of the children are diagnosed. The individual support is videotaped. The supporting lessons are accompanied by a seminar at the university. There, theoretical aspects, reflections on diagnosis and support as well as practical experiences from the individual support are reflexively linked using video-based case studies.

METHOD

The analysis takes place in several steps. In a first step, an interaction analysis is carried out to reconstruct the interaction processes in detail (Krummheuer & Naujok, 1999). In a second step, the diagrammatic activity and the communication about it is analysed. For this purpose, an analysis method developed by Wille (2020) for imagined dialogues was adapted for interactions in the two representation systems twenty field and natural numbers. An analysis sheet is filled in for this purpose (see Figure 3): If a *diagram* is used in a turn, a *filled circle* is set in the column of the corresponding representational system. If *communication about* diagrams is used, a *dashed circle line* is set. If both take place, both are noted together. The filled circles or dashed circle lines are connected to each other by *solid lines* if a connection is made by *diagrammatic activities*. The line is *dashed* when the connection is made by *communicating about diagrams*. If both occur, both are noted together. If, in a turn, *diagrams of different representational systems correspond* with each other, they are connected by an *arrow*. The direction of the arrow indicates which representational system is used as the starting point. Communication that cannot be assigned to either one or the other representation system is noted as “others”.

Activities of the preservice teacher are noted in red, activities of the child in blue.

ANALYSIS

The three scenes analysed in the following are taken from a support situation between a preservice teacher, and a girl who is a student at the beginning of the second grade. In the following, we name them Tom and Samira. The transcripts were originally in German. Tom and Samira work on a task to the part-whole-schema, in which the different arrangements of chips in a row or in a block on the twenty field are to be discussed (see Figure 1).

Figure 1: Thirteen chips arranged in a row (a) with a full upper row of ten and in a block (b) with a full block of ten on the left side on the twenty field

Tom uses a magnetic twenty field with magnetic chips. When a chip is placed, there is a clicking sound. In the transcript, the fields of the twenty field are numbered as shown in Figure 2.

P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
P11	P12	P13	P14	P15	P16	P17	P18	P19	P20

Figure 2: Numbering at the twenty field

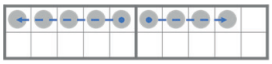
Scene 1

1 Tom: And how would you put the nine now on the field of points? *Pushes the chips together in a pile and closer to Samira.* Do it.

2 Samira: *Places the chips one by one from P1 to P9.*

3 Tom: Exactly. And how can you recognize it‘s nine now? Without counting?

4 Samira: So here five *points from P5 to P1* plus 4 *points from P6 to P9*



5 Tom: *Nods*.

6 Samira: equals nine.

7 Tom: Exactly

Summarizing interpretation

By asking Samira how she would put the nine chips onto the twenty field (turn 1), the preservice teacher Tom suggests that it depends on the arrangement of the nine chips. The arrangement is not given by him but allows Samira to find her own way. Samira places the chips in a row arrangement (turn 2). She begins

on the left in the top line, which indicates that she is familiar with this convention for handling the twenty field. With the question in turn 3 Tom wants to find out, whether Samira can perceive the quantity by structural subitizing. Samira's answer (turn 4 and 6), in which she explains the decomposition of quantities and clarifies it through gestures, seems appropriate for him, as he confirms it (turn 5 and 7). This shows that decomposition seems essential for the preservice teacher.

Semiotic analysis

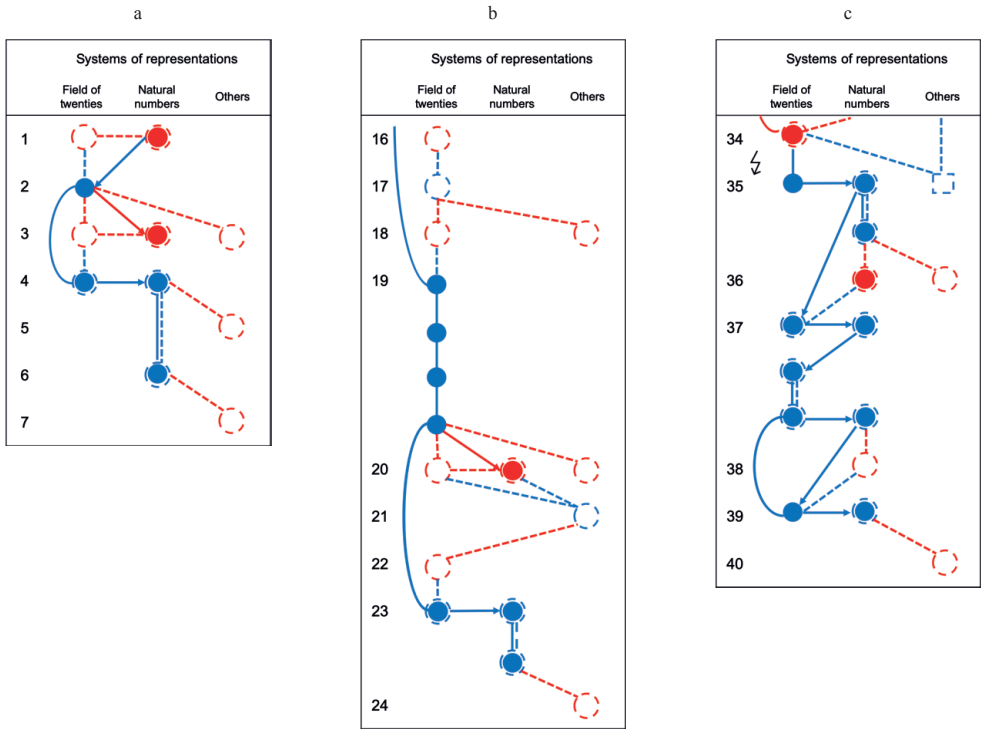


Figure 3: Analysis sheet of scene 1 (a), scene 2 (b) and scene 3 (c)

The semiotic analysis shows that diagrammatic activities take place in both systems of representation (see Figure 3a):

In the *representational system natural numbers*, *diagrams* are used both by Tom and Samira. Samira connects the diagrams in the natural numbers by her own *diagrammatic activities* and *communicates about* them (turn 4-6). Tom uses a diagram for the initiation of the task (turn 1) and for a request in combination with communicating about a diagram in the twenty field (turn 3).

In the *representational system twenty field*, *diagrams* are used exclusively by Samira (turn 2 and 4). Tom only communicates about diagrams in the twenty field. Samira's diagrams are connected by *diagrammatic activity*. In contrast, Samira's diagrammatic activity in the twenty field only takes place upon request of Tom (turn 3). In the same way, *communication about* the diagrams takes

place only upon Tom's request (turn 3) by communicating about the diagrams and establishing a correspondence with the natural numbers (turn 3).

A look at the *correspondences* between the two representational systems shows that Tom creates a correspondence from the twenty field into the natural numbers that Samira has not used (turn 3). Furthermore, Tom initiates that Samira creates a correspondence between the representational systems natural numbers and twenty field (turn 1). In turn 4, Samira creates a correspondence from the twenty field into the natural numbers immediately after Tom created a correspondence with the same direction in turn 3.

Scene 2

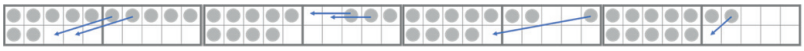
Samira has arranged twelve chips in a row on the field of twenties. This arrangement has been discussed. Then a theoretical repetition of the terms "row" and "block" has been carried out.

16 Tom: What is this now? *Points to the field of twenty.* Row or block?

17 Samira: Row.

18 Tom: Mhm. How would that look as block?

19 Samira: *Moves two chips from P6 and P7 to P13 and P14, two chips from P8 and P9 to P6 and P7, one chip from P10 to P15 and one chip from P7 to P16.*



20 Tom: *Nods slightly.* Mhm . and can you recognize that there are twelve of them real quick?

21 Samira: .. Yes.

22 Tom: Why?

23 Samira: *Points from P1 to P6* six *points from P11 to P16* plus six equals twelve.

24 Tom: Mhm.

Summarizing interpretation

After a repetition of the terms row and block arrangement, and a correct connection of the terms with the present arrangement on the twenty field (turn 16 and 17) Tom wants to find out Samira's abilities with regard to a block arrangement. He therefore asks her to change the present row arrangement, that has been chosen by herself, into a block arrangement (turn 18). Samira reacts by moving chips so that the block of tens on the left side is filled, and all the chips lie on the field as a block (turn 19). The way she moves the chips suggests that Samira does not decide what she wants to do until she is pushing. The question of whether Samira can determine the quantity of twelve by structural subitizing seems again important to Tom (turn 20). He attaches importance to a non-

counting procedure by asking to do it "real quick" (turn 20). Samira hesitates briefly and only affirms his question (turn 21). Either she does not take his question as a request or she does not know the answer or is unsure of the answer. Tom concretizes his question by asking "why" (turn 22), whereupon Samira answers by explaining the decomposition, supported by gestures (turn 23). In her answer, Samira sticks to her focus on the rows by recognizing six in each row.

Semiotic analysis

The semiotic analysis (see Figure 3b) shows many similarities to scene 1. *Diagrammatic activities* take place in both systems of representation. In the *natural numbers*, *diagrams* are used both by the preservice teacher Tom and Samira. Samira connects the diagrams in the natural numbers by her own *diagrammatic activities* and *communicates about* them (turn 4-6). Tom uses a diagram for a request in combination with communicating about a diagram in the twenty field (turn 20).

In the *twenty field*, *diagrams* are used exclusively by Samira (turn 2 and 4). Tom only communicates about diagrams in the twenty field. Samira's diagrams are connected by *diagrammatic activity*. In contrast to the system of natural numbers, her diagrammatic activity in the twenty field takes place upon Tom's request (turn 23) except for turn 19. In turn 19, she connects several diagrams by diagrammatic activity. In the same way, *communication about* the diagrams takes place only upon Tom's request (turn 20, 22), that is made by communicating about the diagrams and establishing a correspondence with the natural numbers (turn 20).

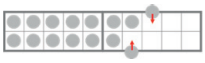
A look at the *correspondences* between the two systems of representations shows that the preservice teacher creates a correspondence to the system of representation that Samira has not used (turn 20), that is a correspondence from the twenty field into the natural numbers. Samira creates a correspondence only from the twenty field into the natural numbers, too (turn 23). These correspondences are created by Tom immediately before (turn 20).

A difference to scene 1 occurs at the very beginning by communicating about diagrams. The next difference occurs in turn 19: on the twenty field several diagrammatic activities are carried out directly one after the other by Samira. From turn 20 to 22 the request for further diagrammatic activities and communicating about them is done in two steps by communicating about diagrams in the twenty field and other communication.

Scene 3

The block arrangement has been discussed with thirteen chips. The arrangement of the twelve as double six from scene 1 was used by pushing the thirteenth chip away and back in again.

34 Tom: *Places two chips audibly on P8 and P17, briefly lifts the chips of P6 and P7 and audibly puts them back in place.* And how many are there now?



35 Samira: *Looks up briefly with the eyes, makes slight nodding movements with the head.* Eight. Äh sixteen.

36 Tom: Mhm. *Swallows.* Why sixteen?

37 Samira: I counted here *points from P1 to P8* that it is eight but there *points from P1 to P17* it can't be eight because there *points to P18* should be one more. So it is not sixteen.

38 Tom: .. But?

39 Samira: *Makes slight nodding movements with the head.* Fifteen.

40 Tom: Mhm.

Summarizing interpretation

By producing more clicking sounds when laying the chips (turn 34), Tom wants to avoid that Samira can continue counting from thirteen on. Samira needs a longer time to determine the cardinality of the quantity (turn 35). The slight nodding movements can be an indication that she is counting. It is also possible that she has counted the clicking sounds and is now confused. When determining the cardinality, she seems to concentrate on the rows, as she first gives the number of chips in the upper row as answer. She corrects herself immediately with another incorrect answer. Samira does not seem to have determined the cardinality of sixteen on the field, but rather to have obtained it by doubling the number of eight in her head. Tom appears surprised by the wrong answer and manages to think for a short time (turn 36). By asking the "why", according to his requests in previous scenes, Tom gets the chance to understand Samira's mistake and gives her the opportunity to justify or revise her answer. Samira takes up this possibility and explains that she has determined the cardinality of eight by counting (turn 37). This, again, shows her concentration on the rows. By comparing the number of chips in the upper and bottom row, she argues with the complete block of sixteen as a double eight. However, she does not seem to be able to determine the correct cardinality immediately, as she does not indicate it. By asking Samira to name the correct result (turn 38) Tom is directly following up on her previous utterance and makes no further comments. Samira, again makes slight nodding movements which indicate a counting procedure (turn 39). This means that she cannot determine the cardinality in the block arrangement without counting. It is also possible that she is still confused by the additional clicking sounds (turn 34) or that she cannot determine the result because she is thinking of an addition task ("there … should be one more" (turn 37)), but would have to subtract.

Semiotic analysis

The semiotic analysis (see Figure 3c) shows, that Samira uses *diagrams* in both representational systems. In the natural numbers as well as in the twenty field she connects diagrams by *diagrammatic activities* or *communicating about it* by herself (turn 35, 37). In turn 37, after the Tom's response to her mistake (↯), Samira establishes *correspondences* from natural numbers to the twenty field. To do this, she uses a diagram from earlier, when the mistake happened. In turn 39, after Tom's *communication about* Samira's diagram in the natural numbers, she creates for the second time a *correspondence* from natural numbers to the twenty field. Tom only uses *diagrams* at the beginning of the task in the twenty field, and one in response to Samira's mistake in turn 36. Here, he remains in the representational system of the natural numbers that she used before. Furthermore, he only *communicates about* the diagrams, whereby he also remains in the system she used before (turn 38) or communicates about other things. This scene is characterized by the fact that Samira switches back and forth between the two systems by herself when using the diagrams.

CONCLUSIONS

In scene 1 and 2 a pattern is visible: In both scenes the diagrammatic activity and communicating about it on the twenty field are stimulated by Tom by a request and a correspondence to the natural numbers (turn 3). The diagrammatic activity in the twenty field consists of gestures (Huth, in press) and shows the possible decomposition for a structural subitizing (Schöner & Benz, 2018). This seems to be essential for Tom and he does not go further into the diagrammatic activity with natural numbers. Changing the arrangement in scene 2 changes almost nothing in the pattern, except Samira's diagrammatic activities in turn 19. This indicates, that the pattern is determined less by the task than by the preservice teacher's requests and Samira's reactions. This is also shown by the fact that a slight reformulation of the request (turn 20) changes the pattern slightly, since the child no longer perceives it as a request for diagrammatic activity.

In scene 3, Samira's error in determining the number of chips and the minimal reaction of Tom in the form of a repetition of her answer as a query (turn 36) causes a change in the pattern. From this moment of irritation (Schülke, 2013) Samira starts to be diagrammatically active by her own, uses both representational systems more flexibly and communicates about it in order to reflect on her previous approach and to find new possibilities. Tom only supports this by asking her to continue her thoughts (turn 38). It can be assumed that the correspondences between the two representational systems carried out by the preservice teacher in scenes 1 and 2 have also contributed to the fact that Samira now also uses correspondences between the two representational systems after the irritation.

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